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METHOD AND ARRANGEMENT FOR DETERMINING A REGULATING VARIABLE OF A TECHNICAL SYSTEM THAT IS DESCRIBED WITH A PREDETERMINED MODEL DESCRIPTION IN A PREDETERMINED SPACE

The invention is directed to a computer-supported determination of a regulating variable of a technical system that is described with a predetermined model description in a predetermined space.

It is known from [1, 2, 3] to employ a continuous model for the description of a system. The following state quantities are employed for describing a state of the system:

– traffic flow velocity v
– vehicle density ρ (ρ = plurality of vehicles Fz/km)
– traffic flow q (q = plurality of vehicles Fz/h], q = v * ρ).

Further a means is known, for example a conductor loop worked into a lane that is coupled to a counter and to an interpretation unit, with which the state quantities (v, ρ, q) of the system of the traffic flow can be measured.

Proceeding from a static relationship between an equilibrium velocity V_{eq} of the traffic flow (V_{eq} = static traffic flow velocity in a stationary state of the traffic flow) and the vehicle density ρ, the model known from [1, 2, 3] describes the traffic flow in an equilibrium state.

The following relationship applies:

$$Veq(\rho) = \sum_{i=1}^{2} w_i \left(1 - \frac{\rho}{\rho_i}\right)^{(l_i - 1)\frac{1}{(1 - m_i)}} \qquad \text{(Equation 1)}$$

with:

w_i or, respectively, ρ_i	: freely selectable imaging parameter
l_i or, respectively, m_i	: freely selectable imaging parameter
i	: run variable
V_{eq} or, respectively, ρ	:equilibrium velocity or, respectively, vehicle density.

It is also known from [1, 2, 3] that both the traffic flow velocity v as well as the vehicle density ρ vary dependent on a location x and on a time t according to the relationship $v = v(x, t)$ or, respectively, $\rho = \rho(x, t)$ [x: location variable, t: time variable].

For describing this dynamic, the model is expanded by a continuity equation (Equation 2) and an acceleration equation (Equation 3).

The continuity equation (Equation 2), corresponding to the relationship

$$\frac{d}{dt}\rho + \frac{d}{dx}q = \frac{d}{dt}\rho + \frac{d}{dx}(\rho v) = 0 \qquad \text{(Equation 2)}$$

with:

q	: traffic flow
d/dt or, respectively, d/dx	: a partial derivation according to the time t or, respectively, according to the location x,

describes the dynamic of the traffic flow under the condition that the traffic flow exhibits a continuous flow without and entry and departure of a vehicle from the system.

The acceleration equation (Equation 3) describes the dynamic of the traffic flow outside the equilibrium state established by the static equilibrium velocity according to Equation 1, using the following relationship:

$$\frac{d}{dt}v + v\frac{d}{dx}v = \frac{1}{\tau}(V_{eq}(\rho) - v) - \frac{c_0^2 d\rho}{\rho dx} + \frac{\eta_0 d^2 v}{\rho dx^2}$$

(Equation 3)

with:

τ	: relaxation time
c_0^2	: velocity variance
η_0	: viscosity constant
d/dt, d/dx, d^2/dx^2	: a partial derivation according to the time t or, respectively, a partial first and a partial second derivation according to the location x.

Proceeding from the description of the traffic flow by such a model, a stability analysis of the model supplies characteristic properties of the traffic flow described by the model.

A local stability analysis of the above-presented model by linearization around a stationary operating point (v_0, ρ_0) shows that the unregulated traffic flow according to the model exhibits an unstable behavior for a vehicle density ρ in a range [approximately 20 Fz/km - approximately 50 Fz/km]. A disturbance of the traffic flow increases and leads to conditions to be observed in real traffic situations such as, for example, a suddenly occurring standstill of the traffic flow (jam) or a "stop-and-go wave".

The system exhibits a stable behavior in the region of the vehicle density ρ [$\rho < 20$ Fz/km] and in the region of the vehicle density ρ [$\rho > 50$ Fz/km].

The following ranges are distinguished:

$\rho < 20$ Fz/km	:	low traffic, high speed, stable behavior
20 Fz/km $< \rho <$ 50 Fz/km		unstable behavior, minor disturbances crop up
$\rho > 50$ Fz/km	:	high traffic volume, slowly moving traffic or jam, stable behavior.

It is also known to apply a method of control technology to a traffic flow model in order to thus assure a regulated and stable traffic flow in the overall state space of the traffic flow.

Realizing a control with a linear state return is known from [4]. The traffic flow in a state wherein the traffic flow exhibits an unstable behavior can thus be stabilized and a uniform flow of the traffic is assured.

The linear approach from [4], however, exhibits various disadvantages. Thus, a stabilization of the traffic flow is only possible for a minor disturbance of the traffic flow or, respectively, only in a small area (Δv, $\Delta\rho$) of the state space (v, ρ, q) around the operating point (v_0, ρ_0) of the linearization. Due, further, to a linear state return, the regulation supplies a regulating variable that, due to the size of its value, cannot be applied to the real traffic flow.

Various method of non-linear control technology are known from [5]. It is also presented in [5] that a structurally variable regulator is used for regulating a non-

linear system due to its ruggedness with respect to a malfunction. The method of equivalent control is applied in [5] for determining the parameters of the structurally variable regulator.

It is also known that a controlled traffic flow model can be utilized for regulating the real traffic flow. To that end, state quantities of a real traffic situation are measured. These state quantities are applied to the control system, whereby the control system determines a regulating variable such as, for example, the traffic flow velocity v_{rated}. Upon employment of a display means such as, for example a changing traffic signal of a traffic guidance system, this regulating variable, a rated velocity according to the above example, is prescribed for the traffic flow.

The invention is based on the problem of specifying a computer-supported method for determining a regulating variable of a technical system, whereby the technical system is stabilized by the regulated system, and whereby the regulating variable can be applied to the technical system.

The problem is solved by the method according to patent claim 1 and by the arrangement according to patent claim 13.

In the method according to patent claim 1, a regulating variable of a technical system is defined, that is described with a predetermined model description in a predetermined space. To that end, the model description is transformed into a sub-space of the space. In this sub-space, a regulator model description is determined from the transformed model description upon employment of a non-linear regulator model. This regulator model description is transformed back into the original space. The regulating variable is determined upon employment of the back-transformed regulator model description.

The arrangement according to patent claim 13 for determining a regulating variable of a technical system that is described with a predetermined model description in a predetermined space comprises a processor that is configured such that the following steps can be implemented:

– transformation of the model description into a sub-space of the space;

– determination of a regulator model description form the transformed model description upon employment of a prescribable non-linear regulator model;

– back-transformation of the regulator model description into the space of the model description;

– determination of the regulating variable upon employment of the back-transformed regulator model description.

What the method and the arrangement achieve is that a regulating variable of a technical system is determined, whereby the controlled technical system stabilizes a disturbance, and that the regulating variable assumes such a value that the regulating variable can be applied to the real system underlying the technical system.

Advantageous developments of the invention derive from the dependent claims.

It is advantageous in one development to utilize the invention for the control of the technical system. A disturbance of the technical system can thus be stabilized, so that the technical system exhibits a stable behavior in the entire state space (v, ρ, q).

In a further development of the invention, the technical system is a traffic flow. It is thus possible to regulate the traffic flow such that a uniform and disturbance-free state of the traffic flow is achieved.

In one development of the invention, it is advantageous to present the traffic flow on the basis of the following relationship:

$$Veq(\rho) = \sum_{i=1}^{2} w_i \left(1 - \frac{\rho}{\rho_i}\right)^{(l_i - 1)\frac{1}{(1 - m_i)}} \qquad \text{(Equation 1)}$$

with:

w_i or, respectively, ρ_i	: freely selectable imaging parameter
l_i or, respectively, m_i	: freely selectable imaging parameter
i	: run variable
V_{eq} or, respectively, ρ	:equilibrium velocity or, respectively, vehicle density.

The above-presented relationship is a suitable model of the real system of the uniform traffic flow and is thus especially suited for the control of the system.

In order to take the location and/or time dependency of the state quantities of a traffic flow into consideration, it is advantageous as a development of the invention to describe the traffic flow with a continuity equation and/or an acceleration equation.

It is advantageous in a development of the invention to present the continuity equation with the following relationship:

$$\frac{d}{dt}\rho + \frac{d}{dx}q = \frac{d}{dt}\rho + \frac{d}{dx}(\rho v) = 0 \qquad \text{(Equation 2)}$$

with:

q	: traffic flow
d/dt or, respectively, d/dx	: a partial derivation according to the time t or, respectively, according to the location x,

and/or to present the acceleration equation with the following relationship:

$$\frac{d}{dt}v + v\frac{d}{dx}v = \frac{1}{\tau}(V_{eq}(\rho) - v) - \frac{c_0^2 d\rho}{\rho dx} + \frac{\eta_0 d^2 v}{\rho dx^2} \qquad \text{(Equation 3)}$$

with:

τ	: relaxation time
c_0^2	: velocity variance
η_0	: viscosity constant
d/dt, d/dx, d^2/dx^2	: a partial derivation according to the time t or, respectively, a partial first and a partial second derivation according to the location x.

The above-presented relationships represent a good model for the location and time dependency of the state quantities of the real system of the traffic flow and are thus especially suited for controlling the system.

In a development of the invention, an especially simple method derives when the transformation into the sub-space of the space is implemented in that a

plurality of dimensions of the space of the space [sic] of the technical system are returned to a dimension of the sub-space.

It is especially advantageous in a development of the invention to describe the non-linear regulator model with a non-linear, structurally variable regulator. The ruggedness in view of a disturbance is thus enhanced, and a good control behavior is assured.

A method of an equivalent regulation is preferably utilized for the design of the non-linear, structurally variable regulator in a development of the invention due to the simple method.

It is especially advantageous to utilize the invention in the framework of a traffic guidance system since a uniform and stable traffic flow of the real system can thus be achieved. The regulating variable and/or a variable that can be defined from the regulating variable can be communicated to a traffic participant with the assistance of a display means for this purpose.

Exemplary embodiments of the invention are shown in Figures 1 through 3 and are explained in greater detail below.

Shown are:

Figure 1 schematic illustration of a real system of a traffic flow;

Figure 2 schematic illustration of the development of a non-linear regulating system for the traffic flow system;

Figure 3 regulation of a real system, traffic flow.

Figure 1 schematically shows a real system of a traffic flow.

Vehicles 102 are moved in a travel direction 106 by their respective drivers 103 on a monitored path segment 101 of a travel path.

State quantities of the system are measured at a predetermined location, a measuring point 104, within the monitored path segment 101.

To this end, a conductor loop 105 is worked into a lane 109, this measuring the plurality i_{Fz} of vehicles 102 that cross the measuring point 104 within a predetermined time span Δt and the respective velocity v_{iFz} of the vehicle 102 that crosses the measuring point 104.

The measured values (i_{Fz}, v_{iFz}) are transmitted to an evaluation unit 107 that is coupled to the conductor loop 105. Dependent on the transmitted quantities, the evaluation unit 107 determines a guideline velocity v_{Rated} 108 that is communicated to the traffic participants upon employment of a traffic guidance system 1120 that is coupled to the evaluation unit 107.

Figure 2 schematically shows the development of a non-linear regulating system for the traffic flow system.

1. Model Description of the Traffic Flow System in the State Space (Step 201)

The model description (step 201) of the traffic flow system in that state space ensues on the basis of:

$$Veq(\rho) = \sum_{i=1}^{2} w_i \left(1 - \frac{\rho}{\rho_i}\right)^{(l_i - 1)\frac{1}{(1-m_i)}} \qquad \text{(Equation 1)}$$

with:

w_i or, respectively, ρ_i	: freely selectable imaging parameter
l_i or, respectively, m_i	: freely selectable imaging parameter
i	: run variable
V_{eq} or, respectively, ρ	:equilibrium velocity or, respectively, vehicle density,

whereby:

$w_1 = 100$ km/h or, respectively, $w_2 = 10$ km/h

$\rho_1 = 100$ Fz/km or, respectively, $\rho_2 = 160$ Fz/km

$l_1 = 3.2$ or, respectively, $l_2 = 2$

$m_1 = 0.9$ or, respectively, $m_2 = 0$ are set.

The following applies for a free velocity v_{free} in the limit value ($\rho \rightarrow 0$): $v_{free} = w_1 + w_2 = 110$ km/h.

$w_1 = 0$ applies for $\rho > \rho 1$ in order to prevent a rise of the $V_{eq}(\rho)$ relationship.

Taking the location and time dependency (x, t) of the state quantity velocity $v = v(x, t)$ and the state quantity $\rho = \rho(x, t)$ into consideration ensues with continuity equation (Equation 2) and acceleration equation (Equation 3):

$$\frac{d}{dt}\rho + \frac{d}{dx}q = \frac{d}{dt}\rho + \frac{d}{dx}(\rho v) = 0 \qquad \text{(Equation 2)}$$

with:

q	: traffic flow
d/dt or, respectively, d/dx	: a partial derivation according to the time t or, respectively, according to the location x,

$$\frac{d}{dt}v + v\frac{d}{dx}v = \frac{1}{\tau}(V_{eq}(\rho) - v) - \frac{c_0^2 d\rho}{\rho dx} + \frac{\eta_0 d^2 v}{\rho dx^2}$$

(Equation 3)

with:

τ	: relaxation time
c_0^2	: velocity variance
η_0	: viscosity constant
d/dt, d/dx, d^2/dx^2	: a partial derivation according to the time t or, respectively, a partial first and a partial second derivation according to the location x.

whereby:

$\tau_0 = 6$ s or, respectively, $c_0 = 13.31$ m/s or, respectively, $\eta_0 = 59.33$ m/s is set.

The effect of a speed limit on the traffic flow is described by a scaling of Equation 1:

$$V_{eq}(\rho, u) = (l+1)V_{eq}(\rho) \qquad \text{(Equation 4)}$$

with:

u	: regulator output quantity
$uV_{eq}(\rho)$	: regulating variable
vfree(l+u)	: displayed maximum speed

2. Transformation of the Model Description into the Sub-Space (Step 202)

For the transformation of the model description into the sub-space, a collective coordinate z (Equation 5) is introduced with:

$z = x - v_s{*}t$ (Equation 5),

whereby v_s indicates the velocity of a solitary wave. This solitary wave is an asymptotic solution of the model equations 1, 2 and 3, said waves having a constant profile and propagating with a constant velocity v_s.

The transformed model description (step 203) (Equation 6) for a solitary wave derives as:

$$\frac{d^2}{dz^2}v + \frac{q_0}{\eta_0}\left(\frac{c_0^2 - (v - v_s)^2}{(v - v_s)^2}\right)\frac{d}{dz}v + \frac{q_0}{\tau\eta_0}\left(\frac{V_{eq}\left(\frac{q_0}{(v - v_s)}, u\right) - v}{(v - v_s)}\right) = 0$$

(Equation 6)

with:

$\frac{d}{dz}v, \frac{d^2}{dz^2}v$: a partial derivation of the first or, respectively, second order of the traffic flow velocity according to the collective coordinate z.

The transformed continuity equation (Equation 7) supplies the constant flow q_0 as secondary condition (Equation 8):

$$v\frac{d}{dz}\rho + \rho\frac{d}{dz}v - v_s\frac{d}{dz}\rho = 0$$ (Equation 7)

$$\rho(v-v_s) = q_0 = \text{const.}$$ (Equation 8).

3. Determination of the Regulating Model Description upon Employment of a Non-Linear, Structurally Variable Regulator (Step 204)

For regulating the transformed model description, a non-linear, structurally variable regulator is utilized [5] on the basis of the control properties.

To that end, the transformed model description (Equation 6) is presented as follows, taking Equation 4 into consideration:

$$\frac{d^2}{dz^2} v = f(v, \frac{d}{dz} v) + b(v, \frac{d}{dz} v)u, \qquad \text{(Equation 9)}$$

$$f(v, \frac{d}{dz} v) = -\frac{q_0}{\eta_0} \left(\frac{c_0^2 - (v - v_s)^2}{(v - v_s)^2}\right) \frac{d}{dz} v + \frac{q_0}{\tau\eta_0} \left(\frac{V_{eq}(\frac{q_0}{(v - v_s)}) - v}{\tau(v - v_s)}\right), \qquad \text{(Equation 10)}$$

$$b(v, \frac{d}{dz} v) = -\frac{q_0}{\tau\eta_0} \left(\frac{V_{eq}(\frac{q_0}{(v - v_s)})}{\tau(v - v_s)}\right), \qquad \text{(Equation 11)}$$

with:

$f(v, dv/dz)$ or, respectively, $b(v, dv/dz)$: imaging rules.

The design of the non-linear, structurally variable regulator ensues upon employment of the method of equivalent regulation [5].

The control rule (Equation 12) reads:

$u = u_e + u_n$ (Equation 12)

with:

u : regulator output quantity

u_e, u_n : equivalent or, respectively, non-continuous part of the regulator output quantity.

The following also applies:

$s = \lambda v + dv/dz$ (Equation 13)

$V_L(s) = (\frac{1}{2})s^2$ (Equation 14)

with:

s : switch variable

λ : system parameter, $\lambda > 0$

V_L : Ljapunow-like function

$V_L(s)$: imaging rule.

The selection of the switch variable s ensues such that the system is stable for s=0 (sliding state).

The regulator output quantity u is determined such that the derivation of the Ljapunow-like function V_L according to the collective coordinate z is negative: $dV_L/dz < 0$. (Equation 15)

The sliding state s = 0 is described in equivalent fashion by disadvantageous/dz=0.

Taking the scaling (Equation 4) and the transformed model description (Equation 6) into consideration, the equivalent part of the regulator output quantity u_e is presented as follows:

$$u_e = \frac{1}{V_{eq}\left(\frac{q_0}{v - v_s}\right)} \cdot \left[v - V_{eq}\left(\frac{q_0}{v - v_s}\right) + + \frac{\tau}{v - v_s}\left[(v - v_s)^2 \left(1 - \lambda \frac{\eta_0}{q_0}\right) - c_0^2 \right] \frac{d}{dz} v \right]$$

(Equation 16)

The non-continuous part of the regulator output quantity u_n is presented as follows:

$$u_n = K \frac{\tau\eta_0 (v - v_s)}{q_0 V_{eq}\left(\frac{q_0}{v - v_s}\right)} \operatorname{sgn}(s) \qquad \text{(Equation 17)}$$

with:

K : system parameter, $K > 0$.

A regulated system in the sub-space is thus obtained (step 205).

4. Back-Transformation of the Regulator Model Description in the State Space of the System (Step 206)

For the back-transformation (step 206), the non-continuous part of the regulator output quantity u_n is neglected.

The back-transformation yields:

$$u_e = \frac{v - V_{eq}(q)}{V_{eq}(q)} + \frac{\tau}{V_{eq}(q)}\left[1 + \lambda \frac{\eta_0}{q_0} - \frac{c_0^2}{(v - v_s)^2} \right] \frac{d}{dt} v \, . \qquad \text{(Equation 18)}$$

Leaving the acceleration term dv/dt out of consideration, this being usually not measured in practice, the following then derives:

$$u_e = \frac{v - V_{eq}(q)}{V_{eq}(q)}. \qquad \text{(Equation 19)}$$

The regulated system 'traffic flow' in the original space of the technical system (step 207) is thus described by the following relationships (Equations 20, 2 and 21):

$$V_{eq}(\rho, u) = (1+u_e)V_{eq}(\rho) = v, \qquad \text{(Equation 20)}$$

$$\frac{d}{dt}\rho + \frac{d}{dx}q = \frac{d}{dt}\rho + \frac{d}{dx}(\rho v) = 0, \qquad \text{(Equation 2)}.$$

$$\frac{d}{dt}v + v\frac{d}{dx}v = -\frac{c_0}{q}\frac{d}{dx}\rho + \frac{\eta_0}{\rho}\frac{d^2}{dx}v. \qquad \text{(Equation 21)}$$

A local stability analysis of the regulated system in the original space exhibits the following properties of the regulated system:
The regulated system exhibits a stable behavior with respect to arbitrary disturbances in the entire state space of the technical system.

The homogeneous and stable state of the regulated system (ρ_{hom}, q_{hom}, v_{hom}) that occurs due to the non-linear and structurally variable control corresponds to the spatially average starting conditions of the system quantities (ρ, q, v).

The regulating variable supplies maximum values (maximum control interventions approximately 25 km/h) that can be applied to the real system 'traffic flow'.

Figure 3 schematically shows how, upon employment of the regulated model of the system 'traffic flow', the real system 'traffic flow' is homogenized.

At a predetermined location 301 of a monitored traffic flow 302, the state quantities (ρ, q, v) of the traffic flow 302 are measured at predetermined time intervals Δt. The measurement is started at prescribable time t = 0s.

The measured starting state quantities of the real system are ρ_{Start}, q_{Start}, v_{Start}.

The measured state quantities (ρ, q, v) are applied to the regulated model of the system. When a disturbance of the real system occurs, the measured state quantities ($\rho_{disturb}$, $q_{disturb}$, $v_{disturb}$) change.

Dependent on the currently supplied state quantities of the system ($\rho_{disturb}$, $q_{disturb}$, $v_{disturb}$) and on the starting state quantities (ρ_{Start}, q_{Start}, v_{Start}), the regulated model determines the regulating variable v_{rated}.

This is displayed to a traffic participant 304 with the assistance of a traffic guidance system 303. At a time t_1, the real system again reaches the stable starting condition (ρ_{Start}, q_{Start}, v_{Start}).

A few alternatives of the invention are indicated below:

One alternative approach for the velocity in the equilibrium is:

$$V_{eq}(\rho) = V_0 ((1 + \exp ((\frac{\rho}{\rho_{max}} - 0.25) / 0.06))^{-1} - (1 + \exp(-0.25 / 0.06))^{-1}$$

whereby $V_0 = 95 km / h$ and $\rho_{max} = 125 Fz / km$ apply.

The acceleration equation can also be replaced with a different approach, insofar as the characteristic properties such as instability in the medium density range and the occurrence of a solitary wave as asymptotic solution are assured.

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